International Journal of Engineering, Science and Mathematics
Vol. 9 Issue 7, July 2020,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

ON SOFT A_RS CONTINUOUS MAPPINGS IN SOFT TOPOLOGICAL SPACES

P. ANBARASI RODRIGO[1], K.RAJENDRA SUBA[2]

1. Assistant Professor , Department of Mathematics
St. Mary's College (Autonomous), Thoothukudi
Manonmaniam Sundaranar University
Abishekapatti, Tirunelveli, India

2. Research Scholar (Full Time), Department of Mathematics
Register number: 1912221209212
St. Mary's College (Autonomous), Thoothukudi
Manonmaniam Sundaranar University
Abishekapatti, Tirunelveli, India

ABSTRACT

In this paper, We introduce a new class of continuous functions called Soft A_RS continuous functions and discuss their relation with various forms of Soft continuous functions. Further We study the characterizations of Soft A_RS continuous functions and reveal the impact of Soft A_RS closure and Soft A_RS interior in those characterizations. Also We establish Soft A_RS irresolute and compare it with Soft A_RS continuous functions.

KeyWords and Phrases: Soft A_RS Closed set, Soft A_RS Open set, Soft A_RS Continuous function, Soft A_RS Irresolute function, Soft A_RS Interior, Soft A_RS Closure.

1. INTRODUCTION

In 1999 Molodstove [5] initiated a new mathematical tool called Soft set Theory to eradicate the inadequacy in parametrization the uncertainity problems. Soft set Theory paved a new pathway to involve the parameters in the framework of the problems arisen with uncertainity.Muhammad Shabir and Naz [9] introduced Soft topological spaces .Meanwhile Aras and Sonmez [1]discussed the properties of Soft continuous mappings. In 2020 ,The authors of this paper [11] paved a new pathway by introducing a new class of generalized closed set called Soft A_RS closed sets in Soft topological spaces. This paper is devoted to Soft A_RS Continuous functions and Soft A_RS Irresolute function. We can extend these theoretical bases to real world applications like information systems, medical diagnosis etc.

2. PRELIMINARIES

In this section, we present the basic definitions and results of Soft set theory which may be found in earlier studies. Throughout this work, X refers to an initial universe, E is a set of parameters, P(X) is the power set of X and A ⊆ E. Throughout this work $(X,\tau,E),(Y,\sigma,K),(Z,\eta,R)$ are Soft topological spaces, Cl(A,E),Int(A,E),SCl(A,E),αCl(A,E) means Soft closure, Soft interior, Soft semi closure, Soft α closure of the Soft set(A,E)respectively.

Definition 2.1: Let (X, τ, E) be a soft topological space. A Soft set (F, E) is called Soft A_RS -Closed set if βcl(F, E) $\tilde{\subseteq}$ Int (U, E) whenever (F, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft ω -open. The set of all Soft A_RS - closed sets is denoted by A_RS C(X).

The respective complements of the above sets are their open forms.

Definition 2.2:[9] Let (X,τ,E) be a Soft topological Spaces over X. The Soft Interior of (F, E) denoted by Int(F, E) is the union of all Soft open subsets contained in (F,E). Clearly Int(F, E) is the largest Soft open set over X which is contained in (F, E).

i) Soft Int(F, E) = $\tilde{\cup}$\{(O.E): (O, E) is Soft open and (O,E) $\tilde{\subseteq}$ (F,E)\}.

ii) Soft Closure of (F, E) denoted by Cl(F,E) is the intersection of Soft closed sets containing (F, E). Clearly Cl(F, E) is the smallest Soft closed set containing (F,E).Cl(F,E) =\{$\tilde{\cap}$ (O. E): (O,E) is Soft closed and (F, E) $\tilde{\subseteq}$ (O,E)\}.

Definition 2.3: A map f: (X,τ,E) → (Y,σ,K) is said to be

1. Soft continuous [3] if inverse image of every Soft open set in (Y,σ,K) is Soft open in (X,τ,E)

2. Soft semi continuous [3]if inverse image of every Soft open set in (Y,σ,K) is Soft semi open in (X,τ,E).

3.Soft α continuous [4] if inverse image of every Soft open set in (Y,σ,K) is Soft α open in (X,τ,E).

4. Soft generalized semi (gs) continuous [7] if inverse image of every Soft open set in (Y,σ,K) is Soft gs open in (X,τ,E).

III. SOFT A_RS CONTINUOUS FUNCTION:

Definition 3.1: A map f: (X,τ,E) → (Y,σ,K) is said to be Soft A_RS continuous if inverse image of every Soft closed set in (Y,σ,K) is Soft A_RS closed in (X,τ,E).

Example 3.2: Let X= {x_1, x_2}, Y = {y_1, y_2} and E= {e_1 , e_2}, K ={k_1, k_2} and f : (X,τ, E) → (Y,σ,K) where τ = { F_1, F_{13}, F_{15}, F_{16}}, A_RSC(X, τ, E) ={ F_1, F_2, F_3 , F_4 ,F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14},F_{15}, F_{16}} and σ = { F_1, F_4, F_7, F_{13}, F_{15}, F_{16}}, A_RSC(Y,σ,K) ={ F_1, F_2, F_3, F_4 ,F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}} is defined as f(F_1) = F_{13}, f(F_2) = F_2, f(F_3) = F_3, f(F_4) = F_4, f(F_5) = F_5, f(F_6) = F_6, f(F_7) = F_9, f(F_8) = F_8, f(F_9) = F_7, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_1, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft A_RS continuous.

Proposition 3.3: The map f: (X,τ,E) → (Y,σ,K) is said to be Soft A_RS continuous if inverse image of every Soft open set in (Y,σ,K) is Soft A_RS open in (X,τ,E).

Proof: Let f: (X,τ,E) → (Y,σ,K) be Soft A_RS continuous and (G,K) be a Soft open set in the Soft topological space(Y,σ,K) . Then f^{-1}((G,K)^c) is Soft A_RS closed in (X,τ,E) and so f^{-1}((G,K)^c)) is Soft A_RS open set in (X,τ,E).

Proposition 3.4: If f: (X,τ,E) → (Y,σ,K) is Soft continuous then it is Soft A_RS continuous .

Proof: Suppose f: (X,τ,E) → (Y,σ,K) is soft continuous. Let (G,K) be an Soft closed set in (Y,σ,K). Since f is Soft continuous, f^{-1} ((G,K)) is Soft open in (X,τ,E). Also we have every Soft closed set is soft A_RS closed. Therefore f^{-1} ((G,K)) is Soft A_RS closed in (X,τ,E). Hence f is Soft A_RS continuous.

Remark 3.5: The converse of the above theorem need not be true.

Example 3.6: In the Soft topological space (X, τ, E), (Y,σ,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y,σ,K) where τ = { F_1, F_{13}, F_{15}, F_{16}}, τ^c ={ F_{14}, F_2, F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_2, F_3 , F_4 ,F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14},F_{15}, F_{16}} and σ = { F_1, F_4, F_7, F_{13}, F_{15}, F_{16}}, σ^c ={ F_4, F_{12}, F_{10}, F_2, F_{15}, F_{16}} then SA_RSC(Y,σ,K) ={ F_1, F_2, F_3, F_4 ,F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}} is defined as f(F_1) = F_{13}, f(F_2) = F_2, f(F_3) = F_3, f(F_4) = F_4, f(F_5) = F_5, f(F_6) = F_6, f(F_7) = F_9, f(F_8) = F_8, f(F_9) = F_7, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_1, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft A_RS continuous. But f^{-1} (F_4) = F_4, f^{-1} (F_{12}) = F_{12}, f^{-1} (F_{10}) = F_{10} Here F_4 F_{12}, F_{10} is not in τ^c of (X,τ, E). Hence f is not soft continuous.

Proposition 3.7: If f: (X,τ,E) → (Y,σ,K) is Soft semi continuous function then it is Soft A_RS continuous .

Proof: Suppose f: (X,τ,E) → (Y,σ,K) is soft semi continuous. Let (G,K) be an Soft closed set in (Y,σ,K). Since f is Soft semi continuous, f^{-1} ((G,K)) is Soft closed in (X,τ,E). Also we have every Soft semi closed set is soft A_RS closed. Therefore f^{-1} ((G,K)) is Soft A_RS closed in (X,τ,E). Hence f is Soft A_RS continuous.

Remark 3.8: The converse of the above theorem need not be true.

Example 3.9: In the soft topological space (X, τ, E), (Y,σ,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y,σ,K) where τ = { F_1, F_{13}, F_{15}, F_{16}}, τ^c ={ F_{14}, F_2, F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_2, F_3 , F_4 ,F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14},F_{15}, F_{16}},SsC(X, τ, E) ={ F_{14}, F_6, F_{10} , F_9 ,F_5, F_4, F_2, F_{15}, F_{16}} and σ = { F_3, F_{11}, F_{15}, F_{16}}, σ^c ={ F_6, F_5, F_{15}, F_{16}} then S A_RSC(Y,σ,K) ={ F_1, F_4 ,F_5, F_6, F_7, F_8, F_9, F_{10}, F_{12}, F_{13},F_{14}, F_{15}, F_{16}} is defined as f(F_1) = F_5, f(F_2) = F_2, f(F_3) = F_3, f(F_4) = F_6, f(F_5) = F_1, f(F_6) = F_4, f(F_7) = F_7, f(F_8) = F_8, f(F_9) = F_9, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_{13}, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft A_RS continuous. But f^{-1} (F_5) = F_1. Here F_1 is not in SsC of (X,τ, E). Hence f is not soft semi continuous.

Proposition 3.10: If f: (X,τ,E) → (Y,σ,K) is Soft $\boldsymbol{\alpha}$ continuous function then it is Soft A_RS continuous .

Proof: Suppose f: (X,τ,E) → (Y,σ,K) is soft α continuous. Let (G,K) be an Soft closed set in (Y,σ,K). Since f is Soft α continuous, f^{-1} ((G,K)) is Soft α closed in (X,τ,E). Also we have every Soft α closed set is soft A_RS closed. Therefore f^{-1} ((G,K)) is Soft A_RS closed in (X,τ,E). Hence f is Soft A_RS continuous.

Remark 3.11: The converse of the above theorem need not be true.

Example 3.12: In the soft topological space (X, τ, E), (Y,σ,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y,σ,K) where τ = { F_1, F_4, F_7 ,F_{13}, F_{15}, F_{16}}, τ^c ={ F_{14}, F_{12},F_{10},F_2,F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_2, F_3 , F_4 ,F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14},F_{15}, F_{16}}, S $\boldsymbol{\alpha}$ C(X, τ, E) ={ F_{14}, F_6, F_{10} , F_9 ,F_5, F_4, F_2, F_{15}, F_{16}} and σ = { F_4, F_7, F_{15}, F_{16}}, σ^c ={ F_{10}, F_{12}, F_{15}, F_{16}} then it is defined as f(F_1) = F_1, f(F_2) = F_2, f(F_3) = F_5, f(F_4) = F_4, f(F_5) = F_3, f(F_6) = F_6, f(F_7) = F_8, f(F_8) = F_7, f(F_9) = F_9, f(F_{10}) = F_{12}, f(F_{11}) = F_{11}, f(F_{12}) = F_{10}, f(F_{13}) = F_{13}, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft A_RS continuous. But f^{-1} (F_{10}) = F_{12} , f^{-1} (F_{12}) = F_{10}. Here F_{12}, F_{10} is not in S α C of (X,τ, E). Hence f is not soft α continuous.

Proposition 3.13: If f: (X,τ,E) → (Y,σ,K) is Soft JP continuous function then it is Soft A_RS continuous .

Proof: Suppose f: (X,τ,E) → (Y,□,K) is soft JP continuous. Let (G,K) be an Soft closed set in (Y,□,K). Since f is Soft JP continuous, f^{-1} ((G,K)) is Soft JP closed in (X,τ,E). Also we have every Soft JP closed set is soft A_RS closed. Therefore f^{-1} ((G,K)) is Soft A_RS closed in (X,τ,E). Hence f is Soft A_RS continuous.

Remark 3.14: The converse of the above theorem need not be true.

Example 3.15: In the soft topological space (X, τ, E), (Y,□,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y,□,K) where τ = { F_1, F_4, F_7 ,F_{13}, F_{15}, F_{16}}, τ^c ={ F_{14}, F_{12},F_{10},F_2,F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_2, F_3 , F_4 ,F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14},F_{15}, F_{16}}, S JPC(X, τ, E) ={ F_2, F_3, F_4 , F_5 ,F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14},F_{15}, F_{16}} and □ = { F_3, F_{11}, F_{15}, F_{16}}, □^{c} ={ F_6, F_5, F_{15}, F_{16}} then it is defined as f(F_1) = F_6, f(F_2) = F_2, f(F_3) = F_3, f(F_4) = F_4, f(F_5) = F_5, f(F_6) = F_1, f(F_7) = F_7, f(F_8) = F_8, f(F_9) = F_9, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_{13}, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft A_RS continuous. But f^{-1} (F_6) = F_1. Here F_1 is not in SJPC of (X,τ, E). Hence f is not soft JP continuous.

Proposition 3.16: If f: (X,τ,E) → (Y,□,K) is Soft A_RS continuous function then it is Soft gsp continuous .

Proof: Suppose f: (X,τ,E) → (Y,□,K) is soft A_RS continuous. Let (G,K) be an Soft closed set in (Y,□,K). Since f is Soft A_RS continuous, f^{-1} ((G,K)) is Soft A_RS closed in (X,τ,E). Also we have every Soft A_RS closed set is soft gsp closed. Therefore f^{-1} ((G,K)) is Soft gsp closed in (X,τ,E). Hence f is Soft gsp continuous.

Remark 3.17: The converse of the above theorem need not be true.

Example 3.18: In the soft topological space (X, τ, E), (Y,□,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y,□,K) where τ = { F_1, F_4, F_7 ,F_{13}, F_{15}, F_{16}}, $τ^c$ ={ F_{14}, F_{12},F_{10},F_2,F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_2, F_3 , F_4 ,F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14},F_{15}, F_{16}}, SgspC(X, τ, E) ={ F_1, F_2, F_3 , F_4 ,F_5, F_6, F_7, F_8,F_9, F_{10}, F_{11}, F_{12}, F_{13},F_{14},F_{15}, F_{16}} and □ = { F_3, F_{11}, F_{15}, F_{16}}, $□^c$ ={ F_6, F_5, F_{15}, F_{16}} then it is defined as f(F_1) = F_1, f(F_2) = F_2, f(F_3) = F_3, f(F_4) = F_4, f(F_5) = F_5, f(F_6) = F_7, f(F_7) = F_6, f(F_8) = F_8, f(F_9) = F_9, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_{13}, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft gsp continuous. But f^{-1} (F_6) = F_7, f^{-1} (F_5) = F_5 . Here F_7 is not in SA_RS C of (X,τ, E). Hence f is not soft A_RS continuous.

Remark 3.19: The concepts of soft A_RS continuous function and soft pre continuous function are independent.

Example 3.20: In the soft topological space (X, τ, E), (Y,□,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y,□,K) where τ = {F_3 ,F_{11}, F_{15}, F_{16}}, $τ^c$ ={ F_6, F_5,F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_4 ,F_5, F_6, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14},F_{15}, F_{16}}, SpC(X, τ, E) ={ F_{14}, F_{13}, F_6, F_{10}, F_9 , F_8 ,F_7, F_5, F_4, F_2, F_1, F_{15}, F_{16}} and □ = { F_1, F_{13}, F_{15}, F_{16}}, $□^c$ ={ F_{14}, F_2, F_{15}, F_{16}} then it is defined as f(F_1) = F_1, f(F_2) = F_4, f(F_3) = F_3, f(F_4) = F_2, f(F_5) = F_5, f(F_6) = F_6, f(F_7) = F_7, f(F_8) = F_8, f(F_9) = F_9, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{14}, f(F_{13}) = F_{13}, f(F_{14}) = F_{12}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft A_RS continuous. But f^{-1} (F_{14}) = F_{12} , f^{-1} (F_2) = F_4. Here F_{12} is not in SpC of (X,τ, E). Hence f is not soft pre continuous.

Example 3.21: In the soft topological space (X, τ, E), (Y,□,K). $X=\{x_1, x_2\}$ $E=\{e_1,e_2\}$, $Y = \{y_1, y_2\}$ and $K =\{k_1, k_2\}$ and f : $(X,\tau, E) \rightarrow$ (Y,□,K) where $\tau = \{F_3 ,F_{11}, F_{15}, F_{16}\}$, $\tau^c =\{ F_6, F_5,F_{15}, F_{16}\}$ then $SA_RSC(X, \tau, E) =\{ F_1, F_4 ,F_5, F_6, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14},F_{15}, F_{16}\}$, $SpC(X, \tau, E) =\{ F_{14}, F_{13}, F_6, F_{10}, F_9 , F_8 ,F_7, F_5, F_4, F_2, F_1, F_{15}, F_{16}\}$ and □ = $\{ F_{14}, F_9, F_{15}, F_{16}\}$, □$^c =\{ F_1, F_8, F_{15}, F_{16}\}$ then it is defined as $f(F_1) = F_2$, $f(F_2) = F_1$, $f(F_3) = F_3$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_6$, $f(F_7) = F_7$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_{10}$, $f(F_{11}) = F_{11}$, $f(F_{12}) = F_{12}$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly f is soft pre continuous. But $f^{-1}(F_1) = F_2$, $f^{-1}(F_8) = F_8$. Here F_2 is not in SA_RSC of (X,τ, E). Hence f is not soft A_RS continuous.

Remark 3.22: The concepts of soft A_RS continuous function and soft g continuous function are independent.

Example 3.23: In the soft topological space (X, τ, E), (Y,□,K). $X=\{x_1, x_2\}$ $E=\{e_1,e_2\}$, $Y = \{y_1, y_2\}$ and $K =\{k_1, k_2\}$ and f : $(X,\tau, E) \rightarrow$ (Y,□,K) where $\tau = \{F_{14} ,F_9, F_{15}, F_{16}\}$, $\tau^c =\{ F_1, F_8,F_{15}, F_{16}\}$ then $SA_RSC(X, \tau, E) =\{ F_1, F_5, F_6, F_8, F_{10}, F_{12}, F_{13},F_{15}, F_{16}\}$, $SgC(X, \tau, E) =\{ F_1, F_3, F_5, F_7, F_8 , F_{11} ,F_{12}, F_{15}, F_{16}\}$ and □ = $\{ F_3, F_{11}, F_{15}, F_{16}\}$, □$^c =\{ F_6, F_5, F_{15}, F_{16}\}$ then it is defined as $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_3$, $f(F_4) = F_4$, $f(F_5) = F_{10}$, $f(F_6) = F_8$, $f(F_7) = F_7$, $f(F_8) = F_6$, $f(F_9) = F_9$, $f(F_{10}) = F_5$, $f(F_{11}) = F_{11}$, $f(F_{12}) = F_{12}$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly f is soft A_RS continuous. But $f^{-1}(F_6) = F_8$, $f^{-1}(F_5) = F_{10}$. Here F_{10} is not in SgC of (X,τ, E). Hence f is not soft g continuous.

Example 3.24: In the soft topological space (X, τ, E), (Y,□,K). $X=\{x_1, x_2\}$ $E=\{e_1,e_2\}$, $Y = \{y_1, y_2\}$ and $K =\{k_1, k_2\}$ and f : $(X,\tau, E) \rightarrow$ (Y,□,K) where $\tau = \{F_{14} ,F_9, F_{15}, F_{16}\}$, $\tau^c =\{ F_1, F_8,F_{15}, F_{16}\}$ then $SA_RSC(X, \tau, E) =\{ F_1, F_5, F_6, F_8, F_{10}, F_{12}, F_{13},F_{15}, F_{16}\}$, $SgC(X, \tau, E) =\{ F_1, F_3, F_5, F_7, F_8 ,F_{11}, F_{12}, F_{15}, F_{16}\}$ and □ = $\{ F_3, F_{11}, F_{15}, F_{16}\}$, □$^c =\{ F_6, F_5, F_{15}, F_{16}\}$ then it is defined as $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_6$, $f(F_4) = F_4$, $f(F_5) = F_1$, $f(F_6) = F_3$, $f(F_7) = F_7$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_{10}$, $f(F_{11}) = F_5$, $f(F_{12}) = F_{12}$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly f is soft g continuous. But $f^{-1}(F_6) = F_3$, $f^{-1}(F_5) = F_{11}$. Here F_3, F_{11} is not in SA_RSC of (X,τ, E). Hence f is not soft A_RS continuous.

Remark 3.25: The concepts of soft A_RS continuous function and soft β continuous function are independent.

Example 3.26: In the soft topological space (X, τ, E), (Y,□,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y,□,K) where τ = {F_3 ,F_{11}, F_{15}, F_{16}}, τ^c ={ F_6, F_5,F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_4, F_5, F_6,F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14},F_{15}, F_{16}}, SβC(X, τ, E) ={ F_{14}, F_{13}, F_6, F_{10}, F_9 , F_8 , F_7 , F_5 , F_4 ,F_2, F_1 ,F_{15}, F_{16}} and □ = { F_{14}, F_9, F_{15}, F_{16}}, $□^c$ ={ F_1, F_8, F_{15}, F_{16}} then it is defined as f(F_1) = F_1, f(F_2) = F_2, f(F_3) = F_3, f(F_4) = F_4, f(F_5) = F_5, f(F_6) = F_6, f(F_7) = F_7, f(F_8) = F_{12}, f(F_9) = F_9, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_8, f(F_{13}) = F_{13}, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft A_RS continuous. But f^{-1} (F_1) = F_1 , f^{-1} (F_8) = F_{12}. Here F_{12} is not in SβC of (X,τ, E). Hence f is not soft β continuous.

Example 3.27: In the soft topological space (X, τ, E), (Y,□,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y,□,K) where τ = {F_{14} ,F_9, F_{15}, F_{16}}, τ^c ={ F_1, F_8,F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_5, F_6, F_8, F_{10}, F_{12}, F_{13},F_{15}, F_{16}}, SβC(X, τ, E) ={ F_{13}, F_6, F_{12}, F_3, F_{10} ,F_8, F_7, F_5, F_4, F_2, F_1, F_{15}, F_{16}} and □ = { F_1, F_4, F_7,F_{13},F_{15}, F_{16}}, $□^c$ ={ F_{14}, F_{12}, F_{10}, F_2, F_{15}, F_{16}} then it is defined as f(F_1) = F_1, f(F_2) = F_8, f(F_3) = F_{14}, f(F_4) = F_{10}, f(F_5) = F_5, f(F_6) = F_6, f(F_7) = F_{12}, f(F_8) = F_2, f(F_9) = F_9, f(F_{10}) = F_4, f(F_{11}) = F_{11}, f(F_{12}) = F_7, f(F_{13}) = F_{13}, f(F_{14}) = F_3, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft β continuous. But f^{-1} (F_{14}) = F_3 , f^{-1} (F_{12}) = F_7, f^{-1} (F_{10}) = F_4, f^{-1} (F_2) = F_8, . Here F_3, F_7, F_4 is not in SA_RSC of (X,τ, E). Hence f is not soft A_RS continuous.

Remark 3.28: The concepts of soft A_RS continuous function and soft ω continuous function are independent.

Example 3.29: In the soft topological space (X, τ, E), (Y,□,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y,□,K) where τ = {F_{14} ,F_9, F_{15}, F_{16}}, τ^c ={ F_1, F_8,F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_5, F_6, F_8, F_{10}, F_{12}, F_{13},F_{15}, F_{16}}, SωC(X, τ, E) ={ F_1, F_3, F_5, F_7, F_8 , F_{11} , F_{12} , F_{15}, F_{16}} and □ = { F_{14}, F_9, F_{15}, F_{16}}, $□^c$ ={ F_1, F_8, F_{15}, F_{16}} then it is defined as f(F_1) = F_6, f(F_2) = F_2, f(F_3) = F_3, f(F_4) = F_4, f(F_5) = F_5, f(F_6) = F_1, f(F_7) = F_7, f(F_8) = F_8, f(F_9) = F_9, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_{13}, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft A_RS continuous. But f^{-1} (F_1) = F_6 , f^{-1} (F_8) = F_8. Here F_6 is not in SωC of (X,τ, E). Hence f is not soft ω continuous.

Example 3.30: In the soft topological space (X, τ, E), (Y,□,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y,□,K) where τ = {F_{14} ,F_9, F_{15}, F_{16}}, τ^c ={ F_1, F_8,F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_5, F_6, F_8, F_{10}, F_{12}, F_{13},F_{15}, F_{16}}, SωC(X, τ, E)

={ $F_1, F_3, F_5, F_7, F_8, F_{11}, F_{12}, F_{15}, F_{16}$} and □ = { F_4, F_7, F_{15}, F_{16}}, □^{c} ={ $F_{10}, F_{12}, F_{15}, F_{16}$} then it is defined as $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_{12}$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_6$, $f(F_7) = F_{10}$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_7$, $f(F_{11}) = F_{11}$, $f(F_{12}) = F_3$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly f is soft ω continuous. But f^{-1} $(F_{10}) = F_7$, f^{-1} $(F_{12}) = F_3$. Here F_3, F_7 is not in SA_RSC of (X,τ, E). Hence f is not soft A_RS continuous.

Remark 3.31: The concepts of soft A_RS continuous function and soft αg continuous function are independent.

Example 3.32: In the soft topological space (X, τ, E), (Y,□,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y,□,K) where τ = {$F_{14}, F_9, F_{15}, F_{16}$}, τ^c ={ F_1, F_8, F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ $F_1, F_5, F_6, F_8, F_{10}, F_{12}, F_{13}, F_{15}, F_{16}$}, S αg C(X, τ, E) ={ $F_1, F_3, F_5, F_7, F_8, F_{11}, F_{12}, F_{15}, F_{16}$} and □ = { $F_5, F_{12}, F_{15}, F_{16}$}, □^{c} ={ $F_{11}, F_4, F_{15}, F_{16}$} then it is defined as $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_3$, $f(F_4) = F_5$, $f(F_5) = F_4$, $f(F_6) = F_6$, $f(F_7) = F_7$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_{11}$, $f(F_{11}) = F_{10}$, $f(F_{12}) = F_{12}$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly f is soft A_RS continuous. But f^{-1} $(F_{11}) = F_{10}$, f^{-1} $(F_4) = F_5$. Here F_5 is not in S αg C of (X,τ, E). Hence f is not soft αg continuous.

Example 3.33: In the soft topological space (X, τ, E), (Y,□,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y,□,K) where τ = {$F_{14}, F_9, F_{15}, F_{16}$}, τ^c ={ F_1, F_8, F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ $F_1, F_5, F_6, F_8, F_{10}, F_{12}, F_{13}, F_{15}, F_{16}$}, S αg C(X, τ, E) ={ $F_1, F_3, F_5, F_7, F_8, F_{11}, F_{12}, F_{15}, F_{16}$} and □ = { $F_1, F_{13}, F_{15}, F_{16}$}, □^{c} ={ $F_{14}, F_2, F_{15}, F_{16}$} then it is defined as $f(F_1) = F_1$, $f(F_2) = F_3$, $f(F_3) = F_2$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_6$, $f(F_7) = F_7$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_{10}$, $f(F_{11}) = F_{14}$, $f(F_{12}) = F_3$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{11}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly f is soft αg continuous. But f^{-1} $(F_{14}) = F_{11}$, f^{-1} $(F_2) = F_3$. Here F_{11}, F_3 is not in SA_RSC of (X,τ, E). Hence f is not soft A_RS continuous.

Remark 3.34: The concepts of soft A_RS continuous function and soft gs continuous function are independent.

Example 3.35: In the soft topological space (X, τ, E), (Y,□,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y,□,K) where τ = {$F_{14}, F_9, F_{15}, F_{16}$}, τ^c ={ F_1, F_8, F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ $F_1, F_5, F_6, F_8, F_{10}, F_{12}, F_{13}, F_{15}, F_{16}$}, S gs C(X, τ, E) ={ $F_1, F_3, F_7, F_8, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}$} and □ = { F_4, F_7, F_{15}, F_{16}}, □^{c} ={ $F_{10}, F_{12}, F_{15}, F_{16}$} then it is defined as $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_3$, $f(F_4) = F_4$, $f(F_5) = F_{10}$, $f(F_6) =$

F_{12}, $f(F_7) = F_7$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_5$, $f(F_{11}) = F_{11}$, $f(F_{12}) = F_6$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly f is soft A_RS continuous. But $f^{-1}(F_{10}) = F_5$, $f^{-1}(F_{12}) = F_6$. Here F_5, F_6 is not in S gs C of (X,τ, E). Hence f is not soft gs continuous.

Example 3.36: In the soft topological space (X, τ, E), (Y,□,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y,□,K) where τ = {F_{14} ,F_9, F_{15}, F_{16}}, τ^c ={ F_1, F_8,F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_5, F_6, F_8, F_{10}, F_{12}, F_{13},F_{15}, F_{16}}, S gs C(X, τ, E) ={ F_1, F_3, F_5, F_7, F_8 , F_{11} , F_{12} , F_{15}, F_{16}} and □ = { F_5, F_{12} ,F_{15}, F_{16}}, □^{c} ={ F_{11}, F_4, F_{15}, F_{16}} then it is defined as $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_3$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_6$, $f(F_7) = F_7$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_{10}$, $f(F_{11}) = F_{11}$, $f(F_{12}) = F_{12}$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly f is soft gs continuous. But $f^{-1}(F_{11}) = F_{11}$, $f^{-1}(F_4) = F_{14}$. Here F_{11}, F_{14} is not in SA_RSC of (X,τ, E). Hence f is not soft A_RS continuous.

Remark 3.37: The concepts of soft A_RS continuous function and soft gp continuous function are independent.

Example 3.38: In the soft topological space (X, τ, E), (Y, □,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y, □,K) where τ = {F_3 ,F_{11}, F_{15}, F_{16}}, τ^c ={ F_6, F_5,F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}}, S gp C(X, τ, E) ={ F_1, F_2, F_4, F_5, F_6, F_7, F_8 , F_9, F_{10}, F_{12} , F_{13} , F_{14} , F_{15}, F_{16}} and □ = { F_1, F_{13}, F_{15}, F_{16}}, □^{c} ={ F_{14}, F_2, F_{15}, F_{16}} then it is defined as $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_3$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_8$, $f(F_7) = F_7$, $f(F_8) = F_6$, $f(F_9) = F_9$, $f(F_{10}) = F_{10}$, $f(F_{11}) = F_{11}$, $f(F_{12}) = F_{12}$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly f is soft A_RS continuous. But $f^{-1}(F_1) = F_1$, $f^{-1}(F_8) = F_6$. Here F_1, F_6 is not in S gp C of (X,τ, E). Hence f is not soft gp continuous.

Example 3.39: In the soft topological space (X, τ, E), (Y, □,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y, □,K) where τ = {F_3 ,F_{11}, F_{15}, F_{16}}, τ^c ={ F_6, F_5,F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}}, S gp C(X, τ, E) ={ F_1, F_2, F_4, F_5, F_6, F_7, F_8 , F_9, F_{10}, F_{12} , F_{13} , F_{14} , F_{15}, F_{16}} and □ = { F_1, F_{13}, F_{15}, F_{16}}, □^{c} ={ F_{14}, F_2, F_{15}, F_{16}} then it is defined as $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_3$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_6$, $f(F_7) = F_7$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_{10}$, $f(F_{11}) = F_{11}$, $f(F_{12}) = F_{12}$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly f is soft gp continuous. But $f^{-1}(F_{14}) = F_{14}$, $f^{-1}(F_2) = F_2$. Here F_2 is not in SA_RSC of (X,τ, E). Hence f is not soft A_RS continuous.

Remark 3.40: The concepts of soft A_RS continuous function and soft strongly g continuous function are independent.

Example 3.41: In the soft topological space (X, τ, E), (Y, □,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y, □,K) where τ = {F_1, F_{13}, F_{15}, F_{16}}, τ^c ={ F_{14}, F_2,F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11},F_{12}, F_{14}, F_{15}, F_{16}}, S strongly gC(X, τ, E) ={ F_2, F_3,F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12} , F_{13} , F_{14} , F_{15}, F_{16}} and □ = { F_5, F_{12}, F_{15}, F_{16}}, □ ^{c} ={ F_{11}, F_4, F_{15}, F_{16}} then it is defined as f(F_1) = F_4, f(F_2) = F_2, f(F_3) = F_3, f(F_4) = F_1, f(F_5) = F_5, f(F_6) = F_6, f(F_7) = F_7, f(F_8) = F_8, f(F_9) = F_9, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_{13}, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft A_RS continuous. But f^{-1} (F_{11}) = F_{11} , f^{-1} (F_4) = F_1. Here F_1 is not in Soft strongly gC of (X,τ, E). Hence f is not soft strongly g continuous.

Example 3.42: In the soft topological space (X, τ, E), (Y, □,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y, □,K) where τ = {F_5 ,F_{12}, F_{15}, F_{16}}, τ^c ={ F_{11}, F_4,F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_3, F_4,F_6, F_7, F_9, F_{11}, F_{13}, F_{14}, F_{15}, F_{16}}, S strongly gC(X, τ, E) ={ F_1, F_2, F_3,F_4, F_6, F_7, F_9, F_{11}, F_{13} , F_{14} , F_{15}, F_{16}} and □ = { F_1, F_{13}, F_{15}, F_{16}}, □ ^{c} ={ F_{14}, F_2, F_{15}, F_{16}} then it is defined as f(F_1) = F_{14}, f(F_2) = F_2, f(F_3) = F_3, f(F_4) = F_4, f(F_5) = F_5, f(F_6) = F_6, f(F_7) = F_7, f(F_8) = F_8, f(F_9) = F_9, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_{13}, f(F_{14}) = F_1, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft strongly g continuous. But f^{-1} (F_{14}) = F_1 , f^{-1} (F_2) = F_2. Here F_1 , F_2 is not in SA_RSC of (X,τ, E). Hence f is not soft A_RS continuous.

Definition 3.43: The map f: (X,τ,E) → (Y,σ,K) is said to be Soft Semi A_RS continuous if inverse image of every Soft semi closed set in (Y,σ,K) is Soft A_RS closed in (X,τ,E).

Example 3.44: In the soft topological space (X, τ, E), (Y, □,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y, □,K) where τ = {F_3 ,F_{11}, F_{15}, F_{16}}, τ^c ={ F_6, F_5,F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_4,F_5, F_6, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16} }and □ = { F_{14}, F_9, F_{15}, F_{16}}, □ ^{c} ={ F_1, F_8, F_{15}, F_{16}}, SsC = { F_5, F_8, F_1, F_{15}, F_{16}} then it is defined as f(F_1) = F_1, f(F_2) = F_2, f(F_3) = F_3, f(F_4) = F_4, f(F_5) = F_5, f(F_6) = F_6, f(F_7) = F_7, f(F_8) = F_8, f(F_9) = F_9, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_{13}, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft semi A_RS continuous.

Definition 3.45: Let (X,τ,E) be a Soft Topological Spaces over X. Then

i) The Soft A_RS Interior of (F, E) denoted by A_RS Int(F, E) is the union of all Soft A_RS open
subsets contained in (F,E).

A_RS Int(F, E) ={ $\tilde{\cup}$ {(O.E): (O, E) is Soft A_RS open and (O,E) $\tilde{\subseteq}$ (F,E)}}.

ii)The Soft A_RS Closure of (F, E) denoted by A_RS Cl(F,E) is the intersection of Soft A_RS closed sets containing (F, E).

A_RS Cl (F,E) =$\tilde{\cap}$ {(O. E): (O, E) is Soft A_RS closed and (F, E) $\tilde{\subseteq}$ (O,E)}.

Definition 3.46: A Soft topological space (X,τ,E) is said to be a Soft T_{A_RS} space if every Soft A_RS closed set is Soft closed.

Definition 3.47: A Soft topological space (X,τ,E) is said to be a Soft S T_{A_RS} space if every Soft A_RS closed set is Soft semi closed.

Definition 3.48: A Soft topological space (X,τ,E) is said to be a Soft α T_{A_RS} space if every Soft A_RS closed set is Soft α closed.

Proposition 3.49: For a subset (A,E) of a Soft topological space (X,τ,E), the following are equivalent.

i. S A_RS O(X,τ,E) is closed under any union.
ii. (A,E) is Soft A_RS closed if and only if A_RS Cl(A,E) = (A,E).
iii. (A,E) is Soft A_RS open if and only if A_RS Int(A,E) = (A,E).

Proof:

1→2: Let (A,E) be a Soft A_RS closed set in the Soft topological space (X,τ,E). Then by definition of Soft A_RS closure, A_RS Cl(A,E)=(A,E).Conversely assume that A_RS Cl(A,E)=(A,E), for each $x \tilde{\in} (A,E)^C$, $x \tilde{\notin}$ PCl(A,E) , therefore there exists a Soft A_RS open set $(G,E)_x$ such that $(G,E)_x \tilde{\cap} (A,E) = \tilde{\emptyset}$ and hence $x \tilde{\in} (G,E)_x \tilde{\subseteq} (A,E)^C$. Therefore $(A,E)^C = \tilde{\cup} (G,E)_x$. Then by (1), $(A,E)^C$ is Soft A_RS open and hence (A,E) is Soft A_RS closed.

2→3: Let (A,E) be a Soft A_RS open set. Then $(A,E)^C$ is a Soft A_RS closed set. Hence, A_RS $Cl((A,E)^C) = (A,E)^C$,by hypothesis, $(A_RS\ Cl((A,E)^C)^C = (A,E)$.That is A_RS int(A,E) = (A,E). converse part of (3) is obvious from converse part of (2).

3→1: let {(U,E)α }:α $\tilde{\in}$∧ } be a family of Soft A_RS open sets of (X,τ,E), Put (U,E) = {$\tilde{\cap}$α (U,E)α}.For each x $\tilde{\in}$ (U,E) there exists α(x) $\tilde{\in}$ U such that x $\tilde{\in}$ (U,E)α(x) $\tilde{\subseteq}$ (U,E).Since x$\tilde{\in}$ (U,E)α(x) is Soft A_RS open , x$\tilde{\in}$ A_RS Int(U,E) and so (U,E) = A_RS Int(U,E). By(3) (U,E) is Soft A_RS open .Then S A_RS O(X,τ,E) is closed under any union.

Theorem 3.50: Let f: (X,τ,E) → (Y,σ,K) be a map. Assume that SA_RS O(X,τ, E) is closed under any union then the following statements are equivalent.

1. The map f is Soft A_RS continuous.
2. The inverse image of each Soft open set in (Y,σ,K) is Soft A_RS open set in (X,τ,E).
3. For each point x $\tilde{\in}$ $\tilde{X}$ and each Soft open set (V,E) in (Y,σ,K) with f(x) $\tilde{\in}$ (V,E) there is Soft A_RS open set (U,E) in (X,τ,E) such that x $\tilde{\in}$ U,E) , f(U,E) $\tilde{\subseteq}$ (V,E) .
4. For each subset (A,E) of (X,τ,E), f(A_RS Cl(A,E) $\tilde{\subseteq}$ Cl(f(A,E))
5. For each subset (B,E) of (Y,σ,K), A_RS Cl(f^{-1} (A,E)) $\tilde{\subseteq}$ f^{-1}Cl(B,E))
6. For each subset (B,E) of (Y,σ,K), f^{-1}(Int(A,E)) $\tilde{\subseteq}$$A_RS$ Int(f^{-1} (B,E))

Proof:

1→2: This is follows from Proposition 3.3

1→3: Suppose that (3) holds and let (G,K) be an Soft open set in (Y,σ,K) and let x $\tilde{\in}$ f^{-1} (G,K).Then f(x) $\tilde{\in}$ (G,K) and thus there exists a Soft A_RS open set $(U,E)_x$ such that x$\tilde{\in}$ $(U,E)_x$ and f($(U,E)_x$) $\tilde{\subseteq}$ (G,K) .Now, x $\tilde{\in}$ $(U,E)_x$ $\tilde{\subseteq}$ f^{-1} (G,K) and f^{-1} (G,K) = $\tilde{x}$ $\tilde{\in}$ (G,K) $\tilde{\cup}$ $(U,E)_x$. By Assumption f^{-1} (G,K) is Soft A_RS open in (X,τ,E) and therefore f is Soft A_RS continuous. Conversely suppose that (1) holds and let f(x) $\tilde{\in}$ (G,K) .Then $x$$\tilde{\in}$ f^{-1} (G,K) in S A_RS O(X,τ,E), since f is Soft A_RS continuous, let (U,E) = f^{-1} (G,K) then $x$$\tilde{\in}$ (U,E) and f(U,E) $\tilde{\subseteq}$ (G,K).

1→4: Suppose that (1) holds and (A,E) be a subset of (X,τ,E) Now (A,E) f^{-1} (f(A,E)) implies (A,E) $\tilde{\subseteq}$ f^{-1} (Cl(f(A,E))) . Since Cl(f(A,E)) is a Soft closed set in (Y,σ,K) ,by assumption, f^{-1} (Cl(f(A,E))) is a Soft A_RS closed set containing (A,E). Consequently, A_RS Cl(A,E) $\tilde{\subseteq}$ f^{-1} (Cl(f(A,E))) .Thus f(A_RS Cl(A,E)) $\tilde{\subseteq}$Cl(f(A,E)) .Conversely suppose that (4) holds for any subset (A,E) of (X,τ,E),Let(G,K)be a closed subset of (Y,σ,K) .Then by assumption, f(S A_RS Cl(f^{-1} (G,K)) $\tilde{\subseteq}$ Cl(f (f^{-1} (G,K))) $\tilde{\subseteq}$ Cl(G,K)=(G,K) .That is , A_RS Cl(f^{-1} (G,K)) $\tilde{\subseteq}$ f^{-1} (G,K) and so f^{-1} (G,K) is Soft A_RS closed in (X,τ,E).

(4) → (5): Suppose that (4) holds (G,K) any Soft subset of (Y,σ,K) replacing (A,E) by f^{-1} (G,K) in (4), then f(A_RS Cl(f^{-1} (B.E))) ⫅̃ f^{-1} (Cl(G,K)) .Conversely, suppose that (4) holds ,let (G,K)=f(A,E),where (A,E) is a Soft subset of (X,τ,E). Then A_RS Cl(A,E) ⫅̃A_RS Cl(f^{-1} (G,K)) ⫅̃ f^{-1} (Cl(f(A,E))) and so f(A_RS Cl(A,E)) = Cl(f(A,E)) .

(5) → (6): Let (G,K) be any subset of (Y,σ,K) then by (5) A_RS Cl(f^{-1} $(G,K)^C$) ⫅̃ f^{-1} (Cl$(G,K)^C$) and Hence (A_RS Int(f^{-1},K))) ⫅̃ (f^{-1} (int(G,K))) Therefore, f^{-1} (int(G,K)) ⫅̃ A_RS Int(f^{-1} (G,K)).

(6) → (1): Suppose (6) holds. Let (G,K) be any closed subset of (Y,σ,K) .Now, f^{-1} ($(G,K)^c$) = f^{-1} (int$(G,K)^C$) ⫅̃ A_RS Int (f^{-1} $(G,K)^c$) = (A_RS Cl(f^{-1} $(F,E))^C$ and hence A_RS Cl(f^{-1} (G,K)) ⫅̃ f^{-1} (G,K) . By Proposition 3.41, f^{-1} (G,K) is Soft A_RS closed .Hence f is Soft A_RS continuous.

4. SOFT A_RS IRRESOLUTE:

Definition 4.1: A map f: (X,τ,E) → (Y,σ,K) is said to be Soft A_RS irresolute if inverse image of every Soft A_RS closed set in (Y,σ,K) is Soft A_RS closed in (X,τ,E).

Example 4.2: In the soft topological space (X, τ, E), (Y, σ,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y, σ,K) where τ = { F_1, F_{13}, F_{15}, F_{16}}, τ^c ={ F_{14}, F_2, F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_2, F_3 , F_4 ,F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14},F_{15}, F_{16}} and σ = { F_2, F_3, F_9, F_{10}, F_{11}, F_{12}, F_{14},F_{15}, F_{16}}, σ^c ={ F_{13}, F_6, F_8, F_7, F_5, F_4, F_1,F_{15}, F_{16}} then SA_RSC(Y, σ,K) ={ F_{13}, F_6, F_8, F_7 ,F_5, F_4, F_1, F_{15}, F_{16}} is defined as f(F_1) = F_1, f(F_2) = F_{13}, f(F_3) = F_3, f(F_4) = F_4, f(F_5) = F_5, f(F_6) = F_6, f(F_7) = F_{12}, f(F_8) = F_9, f(F_9) = F_8, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_7, f(F_{13}) = F_2, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft A_RS irresolute.

Proposition 4.3: The map f: (X,τ,E) → (Y,σ,K) is said to be Soft A_RS irresolute if inverse image of every Soft A_RS open set in (Y,σ,K) is Soft A_RS open in (X,τ,E).

Proof: Let f: (X,τ,E) → (Y,σ,K) be Soft A_RS irresolute and (G,K) be an Soft A_RS open set in the Soft topological space (Y,σ,K) .Then f^{-1} ((G,K)^c) is Soft A_RS closed in (X,τ,E) and so f^{-1} (G,K) is Soft A_RS open set in (X,τ,E).

Proposition 4.4: If a map f: (X,τ,E) → (Y,σ,K) is Soft A_RS irresolute, then it is Soft A_RS continuous.

Proof: Since every Soft open set is Soft A_RS open set, the proof follows.

Remark 4.5: The converse of the above theorem need not be true.

Example 4.6: In the soft topological space (X, τ, E), (Y, σ,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} and K ={k_1, k_2} and f : (X,τ, E) → (Y, σ,K) where τ = { F_1, F_{13}, F_{15}, F_{16}}, τ ^{c} ={ F_{14}, F_2, F_{15}, F_{16}} then SA$_R$SC(X, τ, E) ={ F_1, F_2, F_3 , F_4 ,F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14},F_{15}, F_{16}} and σ = { F_5, F_{12}, F_{15}, F_{16}}, σ ^{c} ={ F_{11}, F_4, F_{15}, F_{16}} then SA$_R$SC(Y, σ,K) ={ F_3, F_4 , F_6, F_7, F_9, F_{11}, F_{13}, F_{14}, F_{15}, F_{16}} is defined as f(F_1) = F_1, f(F_2) = F_2, f(F_3) = F_3, f(F_4) = F_4, f(F_5) = F_5, f(F_6) = F_6, f(F_7) = F_7, f(F_8) = F_8, f(F_9) = F_9, f(F_{10}) = F_{10}, f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_{13}, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is soft A_RS continuous. But f^{-1} (F_7) = F_7, f^{-1} (F_{13}) = F_{13}. Here F_7, F_{13} is not in SA$_R$SC in (X,τ, E). Hence f is not soft A_RS irresolute.

Proposition 4.7 Let (X,τ,E) be a Soft topological space and (Y,σ,K) be a Soft T_{A_RS} space and f: (X,τ,E) → (Y,σ,K) be a map then the following are equivalent.

1. f is Soft A_RS irresolute.
2. f is Soft A_RS continuous.

Proof: ***1*** → ***2*** **:** It follows from Proposition 4.3

2 → ***1*** **:** Let (G,K) be a Soft A_RS closed set in (Y,σ,K) .Since (Y,σ,K) is a Soft T_{ARS} space,(G,K) is a Soft closed set in (Y,σ,K) and by hypothesis, f^{-1} (G,K) is Soft A_RS closed in (X,τ,E). Therefore f is Soft A_RS irresolute.

Theorem 4.8: If the bijective map f: (X,τ,E) → (Y,σ,K) is a Soft ω irresolute and Soft α closed then the inverse map f^{-1}:(Y,σ,K) → (X,τ,E) is Soft A_RS irresolute.

Proof: Let(A,E) be a Soft A_RS closed set in (X,τ,E).Let $(f^{-1})^{-1}$ (A,E) = f(A,E) $\tilde{\subseteq}$ (U,E) , where (U,E) is a Soft ω open in (Y,σ,K) .Then (A,E) $\tilde{\subseteq}$ f^{-1} (U,E) holds. Since f^{-1} (U,E) is Soft ω open in (X,τ,E) and (A,E) is Soft A_RS closed in (X,τ,E),then Scl(A,E) $\tilde{\subseteq}$int(f^{-1} (U,E)) and hence f(Scl(A,E)) $\tilde{\subseteq}$f(int(f^{-1} (U,E)) $\tilde{\subseteq}$int(f(f^{-1} (U,E))) = int(U,E) .Since f is Soft α closed and we know that A map f: (X,τ,E) → (Y,σ,K) is Soft α closed if and only if Scl(f((A,E)) $\tilde{\subseteq}$f(Scl(A,E)) for every subset (A,E) $\tilde{\subseteq}$ $\tilde{X}$. Then, Sclf((A,E)) $\tilde{\subseteq}$f(Scl(A,E)) $\tilde{\subseteq}$Int(U,E) .Thus f(A,E) is Soft A_RS closed in (Y,σ,K) and so f^{-1} is Soft A_RS irresolute.

Theorem 4.9: If the map f: (X,τ,E) → (Y,σ,K) is a Soft ω irresolute, Soft Open, Soft α closed and (A,E) is Soft A_RS closed subset of(X,τ,E)then f(A,E) is Soft A_RS closed in (Y,σ,K) .

Proof: Let(G,K) be an Soft ωopen set in (Y,σ,K) with f (A,E) $\tilde{\subseteq}$ (G,K) .Since f is Soft ω irresolute , f^{-1} ((G,K)) is Soft ω open in (X,τ,E) containing (A,E).Given that (A,E) is Soft A_RS closed therefore Scl(A,E) $\tilde{\subseteq}$int(f^{-1}(G,K)) ,that is f(Scl(A,E)) $\tilde{\subseteq}$ f(int(f(G,K)) $\tilde{\subseteq}$ int(f(f^{-1} (G,K))) = int(G,K) .Since f is Soft α closed, Sclf((A,E)) $\tilde{\subseteq}$f(Scl(A,E)) $\tilde{\subseteq}$Int(A,E) . Thus f(A,E) is Soft A_RS closed.

Proposition 4.10: If the map f: (X,τ,E) → (Y,σ,K) is Soft A_RS irresolute, then f is an Soft irresolute if (X,τ,E) is a ST_{A_RS} space.

Proof: Let(G,K) be Soft semi closed subset of (Y,σ,K) .Then (G,K) is Soft A_RS closed in (Y,σ,K) .Since f is Soft A_RS irresolute, f^{-1} (G,K) is Soft A_RS closed in (X,τ,E). Also since (X,τ,E) is a ST_{A_RS} space, f^{-1} (G,K) is Soft semi closed in (X,τ,E).Then f is Soft irresolute.

5. COMPOSITION THEOREMS:

Remark 5.1: The composition of two Soft A_RS continuous maps need not to be Soft A_RS continuous and this is shown by the following example.

Example 5.2: In the soft topological space (X, τ, E), (Y, σ,K) and (Z, η,R) . X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} , K ={k_1, k_2} and Z = {z_1, z_2}, R={r_1,r_2} and f : (X,τ, E) → (Y, σ,K) , g:(Y,σ,K) → (Z, η,R) where τ = { F_1, F_{13}, F_{15}, F_{16}}, τ^c ={ F_{14}, F_2, F_{15}, F_{16}} then SA_RSC(X, τ, E) ={ F_1, F_2, F_3 , F_4 ,F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14},F_{15}, F_{16}} and σ = { F_5, F_{12}, F_{15},

F_{16}}, σ ^{c} ={ F_{11}, F_4, F_{15}, F_{16}} then SA_RSC(Y, σ,K) ={ F_3, F_4, F_6, F_7, F_9, F_{11}, F_{13}, F_{14}, F_{15}, F_{16}} and η = { F_4, F_7, F_{15}, F_{16}}, η ^{c} ={ F_{10}, F_{12}, F_{15}, F_{16}} is defined as $(g \circ f)$ (F_1) = F_1, $(g \circ f)$ (F_2) = F_2, $(g \circ f)$ (F_3) = F_3, $(g \circ f)$ (F_4) = F_4, $(g \circ f)$ (F_5) = F_5, $(g \circ f)$ (F_6) = F_6, $(g \circ f)$ (F_7) = F_7, $(g \circ f)$ F_8) = F_8, $(g \circ f)$ (F_9) = F_9, $(g \circ f)$ (F_{10}) = F_{10}, $(g \circ f)$ (F_{11}) = F_{11}, $(g \circ f)$ (F_{12}) = F_{12}, $(g \circ f)$ (F_{13}) = F_{13}, $(g \circ f)$ (F_{14}) = F_{14}, $(g \circ f)$ (F_{15}) = F_{15}, $(g \circ f)$ (F_{16}) = F_{16}. Clearly f and g is soft A_RS continuous. But $g \circ f$: (X,τ, E) → (Z, η,R) is not soft A_RS continuous. Since $(g \circ f)^{-1}$ (F_{12}) = F_{13}, F_{13} is not soft A_RS closed in (X,τ, E).Hence gof is not soft A_RS continuous.

Proposition 5.3: If f: (X,τ,E) → (Y,σ,K) is Soft A_RS continuous and g:(Y,σ,K) → (Z, η,R) is Soft continuous $g \circ f$: (X,τ,E) → (Z, η,R) is Soft A_RS Continuous.

Proof: Let (H,R) be a Soft closed set in (Z, η,R) . Since g is Soft continuous then g^{-1} (H,R) is Soft closed in (Y,σ,K) .Since f is Soft A_RS continuous f^{-1} (g^{-1} (H,R)) is Soft A_RS closed set in (X,τ,E).Thus $g \circ f$ is Soft A_RS continuous.

Proposition 5.4: If f: (X,τ,E) → (Y,σ,K) is Contra Soft semi continuous and g:(Y,σ,K) → (Z, η,R) is Contra Soft continuous gof: (X,τ,E) → (Z, η,R) is Soft A_RS Continuous.

Proof: Let (H,R) be a Soft closed set in (Z, η,R) . Since g is Contra Soft continuous then g^{-1} (H,R) is Soft open in (Y,σ,K) .Since f is Contra Soft semi continuous f^{-1} (g^{-1} (H,R)) is Soft semi closed set in (X,τ,E). Since every Soft semi closed set is Soft A_RS closed . $(g \circ f)^{-1}$ (H,R)= f^{-1} (g^{-1} (H,R)) is Soft A_RS closed set in (X,τ,E). Thus gof is Soft A_RS continuous.

Theorem 5.5: If f: (X,τ,E) → (Y,σ,K) and g:(Y,σ,K) → (Z, η,R) be any two maps then

1. $g \circ f$: (X,τ,E) → (Z, η,R) is Soft A_RS Irresolute if both f and g are Soft A_RS irresolute.

2. $g \circ f$: (X,τ,E) → (Z, η,R) is Soft A_RS Continuous if f is Soft A_RS irresolute and g is Soft A_RS continuous.

Proof: 1. Let (H,R) be a Soft A_RS closed set in (Z, η,R) . Since g is Soft A_RS irresolute then g^{-1} (H,R) is Soft A_RS closed in (Y,σ,K) .Since f is Soft A_RS irresolute $(g \circ f)^{-1}$ (H,R)= f^{-1} (g^{-1} (H,R)) is Soft A_RS closed set in (X,τ,E).Thus gof is Soft A_RS irresolute.

2. Let (H,R) be a A_RS closed set in (Z, η,R) . Since g is Soft A_RS continuous then g^{-1} (H,R) is Soft A_RS closed in (Y,σ,K) .Since f is Soft A_RS irresolute $(g \circ f)^{-1}$ (H,R)= f^{-1} (g^{-1} (H,R)) is Soft A_RS closed set in (X,τ,E).Thus $g \circ f$ is Soft A_RS continuous.

6. CONCLUSION

In this paper, we introduced Soft A_RS continuous and Soft A_RS irresolute functions and studied their properties. By suitable Propositions and examples We established the relations between Soft A_RS continuous and other Soft continuous forms. We hope that these findings paved a new pathway to the researchers in this field .This study not only having the theoretical face but also applied in various scenario of real life.

REFERENCES

[1] P.Anbarasi Rodrigo and K.Rajendra Suba, On Soft A_RS Closed sets in Soft Topological Spaces, International Conference on Applied Mathematics and Intellectual Property Rights(ICAMIPR - 2020). (Accepted)

[2] P.Anbarasi Rodrigo and K.Rajendra Suba, On Soft A_RS continuous function in Soft Topological Spaces, International conference on Innovative inventions in Mathematics, Computers, Engineering and Humanities(ICIMCEH - 2020). (Communicated)

[3] P.Anbarasi Rodrigo and K.Rajendra Suba, On Contra Soft A_RS closed sets in Soft Topological Spaces, Advances in Mathematics : Scientific Journal, Vol 9 - 2020.

[4] P.Anbarasi Rodrigo and K.Rajendra Suba, More on Soft A_RS closed sets in Soft Topological Spaces, UGC Care Journal : Purakala journal, Vol 31, 2020, 242 - 248.

[5] P.Anbarasi Rodrigo and K.Rajendra Suba, Soft A_RS open and closed mappings in Soft Topological Spaces, Multidisciplinary Journal ,UGC Care journal , Studies in Indian Place Names, Vol 40, 2020, 2558 - 2566.

[6] P.Anbarasi Rodrigo and K.Rajendra Suba, Functions on Soft A_RS continuous function in Soft Topological Spaces, Khayyam Journal of Mathematics. (Communicated)

[7] C.G. Aras and A. Sonmez, On Soft mappings, arXiv:1305.4545,(2013).

[8] M.Akdag, A.Ozkan, Soft\alpha-open sets and Soft\alpha-continuous functions. Abstr. Anal. Appl. Art ID 891341, 1-7(2014).

[9] H. Aygun and Aygunoglu, Some Notes on Soft topological spaces, Neural Comput. And Applic., Vol.21, No.1(2012), 113-119.

[10] Arockiarani, I. and A. Arokia Lancy, "Generalized Soft gβ-closed sets and Soft gsβ-
closed sets in Softtopological spaces", Int. J. Math. Arch., 4(2): 17- 23,2013.